**Ch 5 - Methods of Proof for Boolean Logic**

**5.7**

**Home(max) | Home(claire)**

**~Home(max) | Happy(carl)**

**~Home(claire) | Happy(carl)**

Home(max) | Home(claire) means that either

* max is home and claire is home
* max is home and claire is not home
* max is not home and claire is home

Assume the first case is true: max is home and claire is home.

Then ~Home(max) is false, and Happy(carl) is true.

Also, ~Home(claire) is false, and Happy(carl) is true.

Therefore Happy(carl) in this case.

Assume the second case is true.

Then ~Home(max) is false, so Happy(carl) must be true.

Also, ~home(claire) is true, so the third premise is true.

Therefore, Happy(carl) in this case.

Assume the third case is true.

Then, ~Home(claire) is false, so Happy(carl) must be true.

The second premise is true because ~Home(max) is true.

Therefore, Happy(carl) is true.

In all three possible cases, Happy(carl) must be true).

**5.8**

**LeftOf(a,b) | RightOf(a,b)**

**BackOf(a,b) | ~LeftOf(a,b)**

**FrontOf(b,a) | ~RightOf(a,b)**

**SameCol(c,a) & SameRow(c,b)**

**BackOf(a,b)**

Either a is left of b or a is right of b.

Case 1: a is left of b, ie Left(a,b) is true

By premise 2, ~Left(a,b) is false, so BackOf(a,b) is true.

Case 2: RightOf(a,b) is true

LeftOf(a,b) must be false, because it is not TW-possible for an object to be to the right and left of another object simultaneously. ~LeftOf(a,b) is true.

By premise 3, ~RightOf(a,b) is false, so FrontOf(b,a) is true.

BackOf(a,b) iff FrontOf(b,a)

Therefore, BackOf(a,b) is true.

In either case, Back(a,b) is true.

**5.9**

**LeftOf(a,b) | RightOf(a,b)**

**BackOf(a,b) | ~LeftOf(a,b)**

**FrontOf(b,a) | ~RightOf(a,b)**

**SameCol(c,a) & SameRow(c,b)**

**LeftOf(b,c)**

Invalid argument.

**5.10**

United Airlines flights from Indianapolis to San Francisco stop at either Denver or Chicago.

Souvenir shops at airport in Chicago sell souvenirs from Chicago Bulls.

Souvenir shops at airport in Denver sell souvenirs from Denver Broncos.

If a flight stops in an airport in a city, and the city has souvenir shops, John has time to stop at the souvenir shop, and buy souvenirs.

Case 1: Flight stops at airport in Chicago.

John has time to stop at the souvenir shop in Chicago.

That souvenir shop sells souvenirs from Chicago Bulls.

John can buy souvenirs from Chicago Bulls.

Case 2: Flight stops at airport in Denver.

John has time to stop at the souvenir shop in Denver.

That souvenir shop sells souvenirs from Denver Broncos.

John can buy souvenirs from Denver Broncos.

Therefore, in case 1 John buys souvenirs from Chicago Bulls, one of Max’s favorite teams, and in case 2 John buys souvenirs from Denver Broncos, one of Max’s favorite teams. In either case, John buys souvenirs from one of Max’s favorite teams.

**5.12**

Rational(n) | ~Rational(n) is a tautology. This sentence has the form A | ~A, and only one of A or A can be true. Therefore, we have two mutually exclusive cases to consider. For n = sqrt(2)^sqrt(2), n can be either rational or irrational on the basis of the previous reasoning.

**5.13**

We will have a maximum sitting capacity of either 6, 8, or 10 places for people to sit at our dinner table.

We will either have a bench on one side and chairs on the other, or chairs on both sides.

We will either have a chair at each end or no chairs at each end.

If we have a bench we can sit five people on the bench.

If a side of the table contains chairs, it must contain exactly three chairs.

Comfort is measured from 0 to 10.

Each black chair has comfort z.

Each bench position has comfort 3.

Non-black chairs have comfort y.

Average comfort level has to be x, where average comfort is measured as sum of comfort levels of all sitting positions, divided by number of seats

* Case 1 bench on one side chairs on the other
  + Case 1.1 Chairs on each end
    - sitting capacity is 10: 5 chairs, 5 bench positions
  + Case 1.2 no chairs on each end
    - sitting capacity is eight: 3 chairs, and five on the bench
    - average comfort is determined by considering the four possible combinations of types of chairs: bbb, bbn, bnn, nnn: 3z, 2z+y, z+2y, 3y
* Case 2 chairs on both sides
  + Case 2.2.1 Chairs on each end
    - sitting capacity is eight
    - average comfort is determined by considering the possible combinations of both types of chairs: 2^7 = 128
  + Case 2.2.2 no chairs on each end
    - sitting capacity is six: 3 chairs on each side

**5.14**

S tautological consequence of P

S tautological consequence of Q

S tautological consequence of P | Q

First let’s note that if P is true then S is true, and if Q is true then S is true. If P is false, we don’t know the truth value of S, and the same goes for Q and S.

P | Q is true iff (P & Q) | (P & ~Q) | (~P & Q)

If (P & Q) is true then both P and Q are true. Therefore, S is true.

If (P & ~Q) is true, then because P is true, S is true. Because ~Q is true, Q is false.

**5.15**

**b is a tetrahedron.**

**c is a cube.**

**Either c is larger than b or else they are identical**

**b is smaller than c**

Let’s first prove that ~(b=c) using a proof by contradiction.

Assume b = c. Then by identity elimination, b is a cube. This contradicts the premise that b is a tetrahedron. Therefore, Cube(b) is false, so b = c is false. This follows because b=c, Cube(c) imply Cube(b). So if Cube(b) is false, then since Cube(c) is true, b=c must be false.

Therefore c is larger than b. Therefore b is smaller than c.

**5.16**

**(Home(max) | Home(claire) & (~Happy(scruffy) | ~Happy(carl))**

**~Home(max) | Happy(carl)**

**~Home(claire) | ~Happy(scruffy)**

**~Happy(scruffy)**

((Home(max) | Home(claire)) & ~Happy(scruffy)) | ((Home(max) | Home(claire)) & ~Happy(carl))

(~Happy(scruffy) & Home(max)) | (~Happy(scruffy) & Home(claire)) | (~Happy(carl) & Home(max)) | (~Happy(carl) & Home(claire))

**Case 1:** ~Happy(scruffy) & Home(max)

by conjunction elimination, ~Happy(scruffy)

**Case 2:** ~Happy(scruffy) & Home(claire))

by conjunction elimination, ~Happy(scruffy)

**Case 3:** ~Happy(carl) & Home(max)

~Home(max) is false, so Happy(carl) is true.

But by assumption ~Happy(carl). Our hypothetical case implies a false statement. Therefore, the hypothetical case must be false, ie a contradiction. It’s negation is always true.

~(~Happy(carl) & Home(max)) logically equivalent to ~~Happy(carl) | ~Home(max) logically equivalent to Happy(carl) | ~Home(max)

In fact, our case assumption is the negation of premise 2, so it must be false. This case is thus not logically possibly.

**Case 4:** Happy(carl) & Home(claire)

~Home(claire) is false so ~Happy(scruffy) is true.

In all logically possible cases, ~Happy(scruffy) is true. Therefore ~Happy(scruffy) is a logical consequence of the premises, and the argument is valid.

**5.17**

**Case 1: Cube(a) is true**

~Cube(a) is false, therefore **(a=b) | Large(a)**

**Case 1.1 a = b is true**

Therefore, a = b | a = c is true

**Case 1.2 Large(a) is true**

~Large(a) is false, therefore **a=c is true.** Therefore a = b | a = c is true

fourth premise is logically equivalent to ~(c=c) | ~Tet(a)

~(c=c) is logically impossible, therefore **~Tet(a) is true**.

**Case 2: Tet(a) is true**

4: c = c is false, a logical impossibility. Therefore, this case is logically impossible.

**Case 3: Large(a) is true**

~Large(a) is false, so a = c is true.

Therefore a = c | a = b is true.

In all logically possible cases, the conclusion is true when the premises are true.

**5.18**

Premise four tells us that ~(c=c) | ~Tet(a). Therefore ~Tet(a) must be true. This result plus premise one means that Cube(a) | Large(a)

**Case 1: Cube(a) is true**

~Cube(a) is false.

Therefore, a=b | Large(a).

Together with premise 3 this means (a=b | Large(a)) & (~Large(a) | a=c)

((a=b | Large(a)) & ~Large(a)) | ((a=b | Large(a)) & a=c)

Therefore (a=b | Large(a)) & a=c)

Therefore (a=c & a=b) | (a=c & Large(a))

Case 1.1 a=c & a=b

Hence, a=b=c

**Case 1.1: ~Large(a)**

Conjunction introduction: ~Large(a) & ~Tet(a) therefore ~(Large(a) | Tet(a))

**Case 1.2: Large(a)**

Assume ~Large(a) | ~Tet(a)

Case 1: ~Large(a)

Cube(a) | Tet(a)

~Cube(a) | a=b

Therefore (Cube(a) | Tet(a)) & (a=b)

But we know that ~Tet(a) is true, so Cube(a) is true.

**5.19**

Owned(claire, folly, 2) | Owned(claire, folly, 2:05)

Owned(max, folly, 2)

Owned(claire, folly, 2:05)